

Pascal's Triangle and Binomial Expansion Examples

This triangle was studied by *Blaise Pascal* and *Omar Khayyam*, and was described about 500 years earlier by the Chinese mathematician, *Yang Hui*.

The pattern for binomial expansion using Pascal's Triangle is given below.

$$(x + y)^0 = 1$$

$$(x + y)^1 = 1x + 1y$$

$$(x + y)^2 = 1x^2 + 2xy + 1y^2$$

$$(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$$

$$(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

and so on....

For example:

$$1. \quad (3x + 2)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$$

substitute for x

$$= 1(3x)^3 + 3(3x)^2(2) + 3(3x)(2)^2 + 1(2)^3$$

substitute for y

$$= 1 \cdot 27x^3 + 3(9x^2)(2) + 3(3x)(4) + 1 \cdot (8)$$

$$= 27x^3 + 54x^2 + 36x + 8$$

Pattern

Substitute

Simplify

Simplify

$$2. \quad (x + 3)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

substitute for y

$$= 1x^4 + 4x^3(3) + 6x^2(3)^2 + 4x(3)^3 + 1(3)^4$$

$$= 1 \cdot x^4 + 4x^3(3) + 6x^2(9) + 4x(27) + 1 \cdot (81)$$

$$= x^4 + 12x^3 + 54x^2 + 108x + 81$$

Pattern

Substitute

Simplify

Simplify

$$3. \quad (3x - 4y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

substitute for x

$$= 1(3x)^4 + 4(3x)^3(-4y) + 6(3x)^2(-4y)^2 + 4(3x)(-4y)^3 + 1(-4y)^4$$

substitute for y

$$= 1 \cdot 81x^4 + 4(27x^3)(-4y) + 6(9x^2)(16y^2) + 4(3x)(-64y^3) + 1 \cdot 256y^4$$

$$= 81x^4 - 432x^3y + 864x^2y^2 - 768xy^3 + 256y^4$$

Pattern

Substitute

Simplify

Simplify

Pascal's Triangle and Binomial Expansion Activity

Use a colored pencil or marker to color each of the following on separate Pascal's Triangles:

1. Even numbers (#s divisible by 2)
2. multiples of 3 (#s divisible by 3)
3. multiples of 4 (#s divisible by 4)
4. multiples of 5 (#s divisible by 5)
5. multiples of 6 (#s divisible by 6)
6. multiples of 7 (#s divisible by 7)

Can you find any other divisibility patterns?

Use Pascal's Triangle and the Binomial Expansion Pattern (on the reverse side) to expand the following Binomials.

1. $(x+4)^2$
2. $(x-4)^3$
3. $(x+4)^4$
4. $(x+4)^5$
5. $(2x+3)^3$
6. $(2x-3)^4$
7. $(2x+3)^5$
8. $(3x-2y)^3$
9. $(3x+2y)^4$
10. $(3x-2y)^5$
11. $(2x+y)^6$
12. $(3x-y)^7$
13. $(x-2y)^8$
14. $(x+2y)^{10}$